Class: B.A/ B.Sc./ BCA M.Marks: 80
Semester: IV Time allowed: 3hours

Subject: Mathematics

Note: Attempt five questions selecting one from each unit. All carry equal marks.

UNIT-I

1. (a) Define interior point of a set, and prove that a subset A of the real line is open if and only if A = A^o.
   (b) Show that the union of a finite family of compact sets is compact. (6,6)

2. (a) Define closure of a set and show that a set A is closed if and only if A = $\bar{A}$.
   (b) Show that every infinite and bounded subset of the real line has a limit point. (6,6)

UNIT-II

3. (a) State completeness property of reals. Is the set of rational numbers complete? Justify your answer.
   (b) Define a monotone sequence. Also prove that a monotonically increasing sequence converges if and only if it is bounded above.
   (6,6)

4. (a) Define a Cauchy sequence. Also show that a Cauchy sequence is always convergent.
   (b) Show that the sequence $\{\frac{2n-9}{3n+1}\}$ is bounded, monotonically increasing and has the limit 2/3. (6,6)

UNIT-III

5. (a) Define convergence and divergence of a positive term series. Also show that the series $\sum a_n$ is convergent implies $\lim_{n\to\infty} a_n = 0$. Is its converse true? Explain.
(b) Show that the series: $1.2/(3^2.4^2) + 3.4/(5^2.6^2) + 5.6/(7^2.8^2) + ...$ is convergent. (6,6)

6. (a) Test the convergence or divergence of the series:
$2x/1 + 3^2x^2/2^3 + 4^3x^3/3^4 + ...$, where $x>0$.
(b) State and prove Ratio test for positive terms series. (6,6)

UNIT-IV

7. (a) Define continuity of a function at a point. Also find the value K so that the function $f(x) = \begin{cases} (x^2-25)/(x-5), & \text{if } x\neq 5 \\ K, & \text{if } x=5 \end{cases}$
is continuous at x=5.
(b) If f is a continuous function on [a, b], then show that it is bounded. (6,6)

8. (a) Show that the function $f(x) = x+1$ is uniformly continuous on [1, 2].
(b) If a function is uniformly continuous on an interval, then show that it is continuous. Is the converse of the result true? Explain. (6,6)

UNIT-V

9. (a) Define derivative of a function at any point. Also examine the derivability of f(x) at x=2, where $f(x) = \begin{cases} 3-2x, & x<2 \\ 3x-7, & x\geq 2 \end{cases}$
(b) State Lagrange's mean value theorem and verify it for $f(x) = x^3+x^2-6x$ in [-1, 4]. (6,6)

10. (a) State and prove Taylor's theorem with Cauchy's remainder.
(b) Expand $f(x) = \tan^{-1}x$ as a series in x up to x^5. (6,6)