B.A/ B.sc/ B.com Examination Semester-IV

Mathematics

(Abstract Algebra)

CBCS

Time Allowed: 3hours M.M:120

Note: The question paper consists of three sections.

Section-A

All questions are compulsory. Each question carries 6 marks

1. Show that the set of all matrices of the form $\begin{pmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{pmatrix}$ where α is a real number, forms an abelian group under the operation of matrix multiplication.
2. Prove that intersection of two sub groups of group is again a subgroup of the group.
3. Let H and K be two sub group of a group G. If H is a normal subgroup of G, then prove that HK = KH is a subgroup of G.
4. Prove that the set I of all integers is a ring with respect to usual addition and multiplication of integers.
5. Let R_1 and R_2 be two rings and S_1 and S_2 be two sub rings of R_1 and R_2 respectively. Then prove that S_1x S_2 is a sub ring of R_1xR_2.

Section-B

All questions are compulsory. Each question carries 10 marks

6. Show that the set of all permutations on a non-empty set forms a group under the composition of maps.

7. If H and K are finite subgroups of a group G, then prove that

$$O(HK) = \frac{O(H).O(K)}{O(H\cap K)}$$

8. State and prove Euler's theorem.

9. Prove that the set R={(a, b) /a, b∈R} is a commutative ring under the addition and multiplication of ordered pairs defined as:

$(a, b) + (c, d) = (a+c, b+d)$

& $(a, b)(c, d) = (ac, bd)$ for all $(a, b), (c, d) \in R$

10. Prove that an ideal P of a commutative ring R is prime if and only if R/P is an integer domain.

Section-C

Attempt any two questions. Each question carries 20 marks.

11. i) Let $(G,*)$ be a group, then show that $(a*b)^{-1} = (b^{-1}*a^{-1})$ for all $a, b \in G$. (08)

ii) Let G be a finite group and let $a \in G$ be an element of order n. Then prove that $a^m=e$, if and only if n is a divisor of m. (12)

12. i) Prove that a non-empty subset H of a group G is a sub group iff

a) $ab \in H$ for all $a, b \in H$

b) $a^{-1} \in H$ for all $a \in H$ (12)

ii) If G be a cyclic group of order n, then prove that for any divisor m of n, there exists a unique subgroup of G of order m. (8)

13. Define Normal subgroup of a group G. Let H be a subgroup of a group G. Then prove that the following statements are equivalent:

i) $ghg^{-1} \in H$ for all $g \in H, h \in H$

ii) $gHg^{-1} = H$ for all $g \in H$

iii) $gH = Hg$ for all $g \in H$

14. i) Define the terms: Ring, Integral domain, Division Ring and field with one example each.

ii) Show that the set of rational numbers Q is a field under the composition $\oplus$ and $\odot$ defined as:

$a \oplus b = a+b-1$ and $a \odot b = a+b-ab$ for all $a, b \in Q$

15. i) Prove that intersection of two ideals of a ring R is an ideal of R. Is union of two ideals of a ring R an ideal of R? Justify your answer.

ii) State and prove Fundamental theorem of Ring homomorphism.