GOVT. COLLEGE FOR WOMEN PARADE GROUND, JAMMU

B.A/B.SC/BCA-III Semester(CBCS)

C.NO. UMATC-301 **SUBJECT: Mathematics**

Time: 3 hours **Marks-120**

Course Title: Real Analysis

NOTE: The question paper is divided into three sections A,B and C. Section A consists of 5 questions of 6 marks each, section B consists of 5 questions of 10 marks each and section C consists of 5 questions of 20 marks each. All the questions of sections A and B are compulsory. Attempt any two questions from section C.

SECTION-A

Q-1 Find the l.u.b and g.l.b of the set

$S=\{-\sqrt{1-x^2} : |x| \leq \frac{\sqrt{3}}{2}\}$

Q-2 Prove that a convergent sequence is bounded.

Q-3 Show that the alternating series $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+$-----is convergent

Q-4 By definition show that $\{f_n\}$,where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in any interval [0,b], b>0.

Q-5 Find the radius of convergence of the power series $\sum (1+\frac{1}{n})^{n^2} x^n$.

SECTION-B

Q-6 Prove that the set N×N is denumerable.

Q-7 Prove that the sequence $\{\frac{2n-7}{3n+2}\}$ is

(i) Monotonically increasing
(ii) Bounded
(iii) Convergent.

Q-8 Discuss the convergence or divergence of the series:

$\sum (\sqrt{n^4+1} - \sqrt{n^4-1})$

Q-9 Test the sequence $\{f_n(x)\}$, where $f_n(x) = \frac{nx}{1+n^3x^2}$ for uniform convergence on [0,1] using M_n-Test.

Q-10 State and Prove Weierstrass's M-Test for uniform convergence of a series of function.

SECTION-C

Q-11 (a) State and prove Archimedian property of Real number.

(b) Prove that between two distinct real numbers ,there exists a rational number and hence infinitely many rational numbers.

Q-12 (a) Prove that a Cauchy sequence is always convergent in R

(b) State and prove Nested Interval Theorem.

Q-13 (a) Prove that the series $\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} +$-------

Converges if p>1 and direrges if o<p≤ 1.

(b) Discuss the convergence of the series

$\frac{x^2}{2\sqrt{1}} + \frac{x^3}{3\sqrt{2}} \frac{x^4}{4\sqrt{3}}$ +------,x>o

Q-14 State and prove M_n-Test for uniform convergence of a sequence of functions.

Q-15 (a) Prove that the series $\sum_{n=0}^{\infty} (-1)^n x^n$, $0 \leq x < 1$)is term by term integrable.

(b) Show that the series $\sum \frac{1}{n^2+n^4x^2}$ is term by term differentiable.